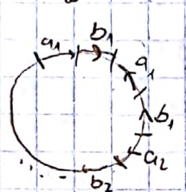
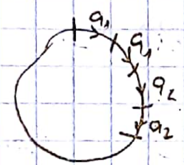
Lecture 3: (from next sunday · Shreiber 007) only on sundays. 11/3/2017

Thm.:

Classification of connected closed surfaces (Dehn-Heegard 1907):

Every closed surface is one of the following.

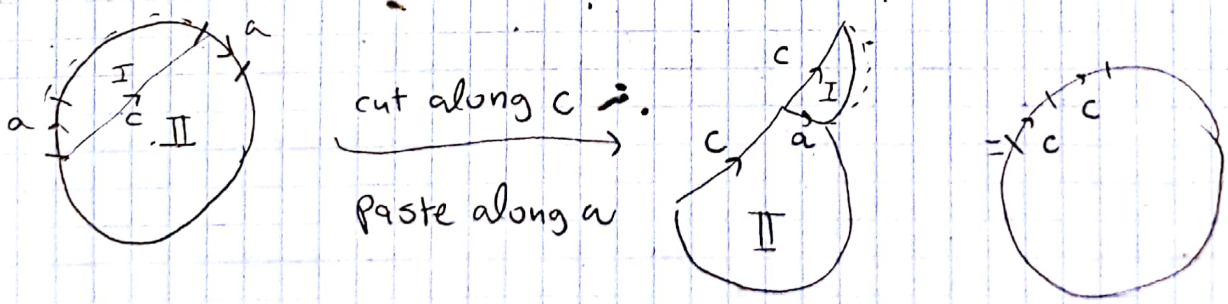
	Canonical form	E.C	orientability	π_1
1) Sphere S^2	$a \bigcirc a$	2	$+$	$\{e\}$
2) $\underbrace{T\#T\#\ \#T}_{n}$		$2-2n$	$+$	$\langle a_1,b_1,\dots,a_n,b_n \mid [a_1,b_1]\dots[a_n,b_n]\rangle$
3) $\underbrace{P\#\ \ \#P}_{n}$		$2-n$	$-$	$\langle a_1,\dots,a_n \mid a_1^2a_2^2\dots a_n^2\rangle$

Why every connected closed surface is homeomorphic to one of these?

Step 1: Reduction to a single equivalence class of vertices.

Step 2: Crosscap normalization:

All pairs of like oriented edges replaced by adjacent pairs.



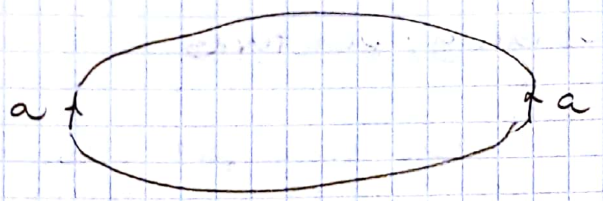
Step 3: Handle normalization.

Claim:

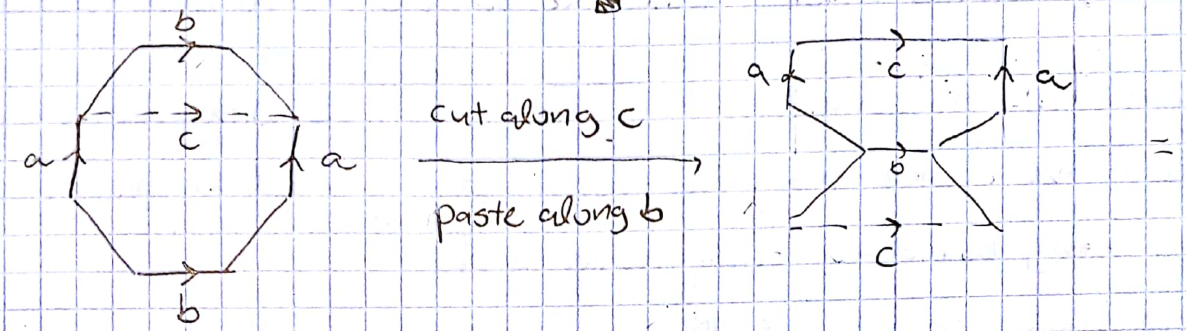
Every pair of opposite oriented edges must occur as "crossed" pairs.

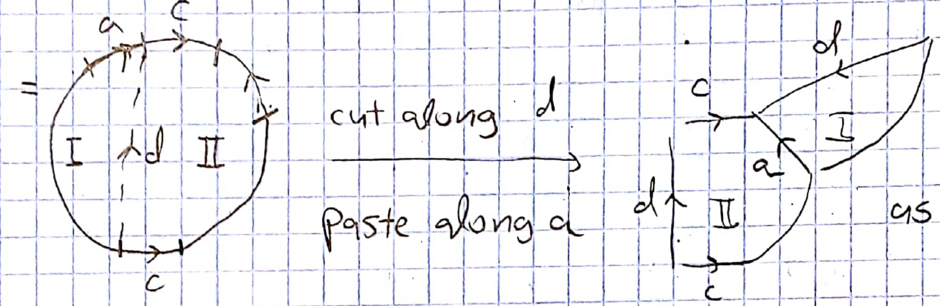
$$\dots a \dots b \dots a^{-1} \dots b^{-1} \dots$$

Pf.:



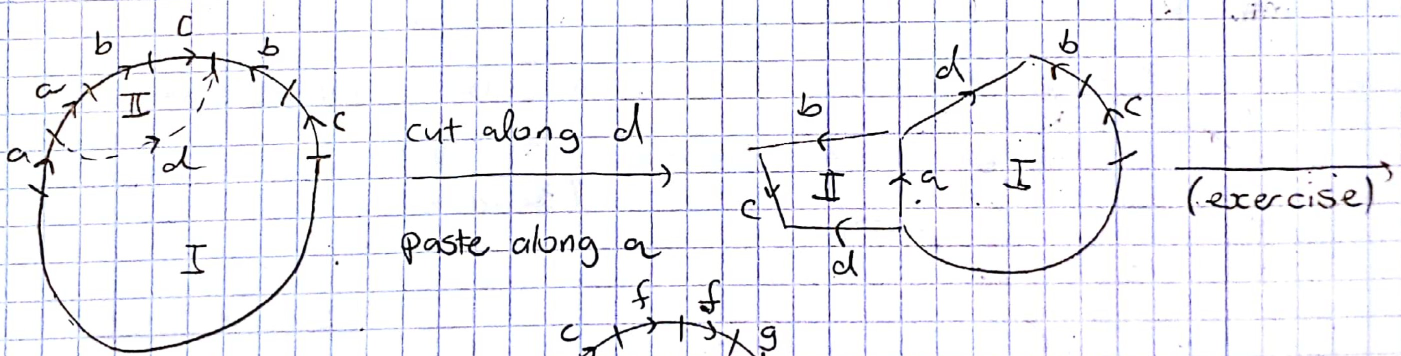
Otherwise, the upper vertices are not glued with the bottom ones.

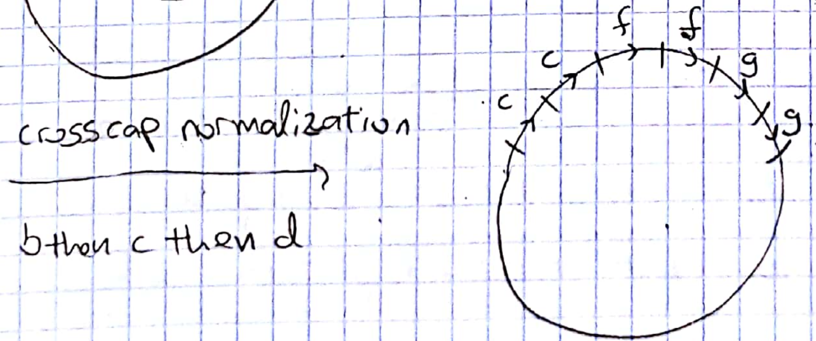




here $c^{-1}dcd^{-1}$ as a subword.

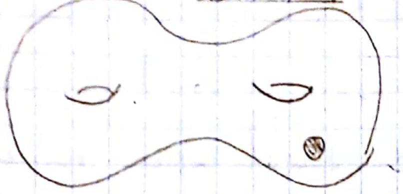
Step 4: If both crosscaps & handles $\Rightarrow$ only crosscaps.





Surfaces with boundary / punctures

boundary



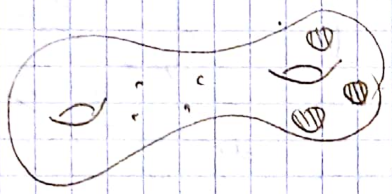
where every point looks localy, either like $\mathbb{R}_2$ or like :

Punctures: points removed from the surface. $\Rightarrow$ no longer compact.

classification of connected compact surfaces with punctures:

On top of the choice from the above list, we need to specify

1) how many boundary components $\in \mathbb{N}$
2) how many punctures $\in \mathbb{N}$



Note:

All compact surfaces with a boundary can be embedded into $\mathbb{R}^3$.