

Planned Topics:

- 1) Classification of surfaces (John Stillwell/Classical topology & combinatorial gr. theory, John Lee/Intro to Topological manifolds)
- 2) The arc complex (Harer) (Hatcher 1991)
- 3) Surfaces & commutator length (Culler 1981)
- 4) Random Gaussian Matrices & enumeration of Graphs on Surfaces. (Zando-zvonkin/graphs on surfaces & Applications (chapter 3))
- 5) Mapping Class Groups (MCG - A primer (Farb-Margalit))
- 6) Random Unitary Matrices & surfaces (Maye-p)

Lecture 1:

Classification of Surfaces:

Def:

A surface is a 2-manifold. i.e. a topological space which is:

- 1) locally Euclidean of dimension 2
- 2) Hausdorff. (counter example - plane with two origins)
- 3) Second countable. Countable basis to the topology compact, no boundary

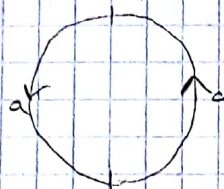
Examples: (to closed surfaces) & connected



the 2-sphere
 S^2



T



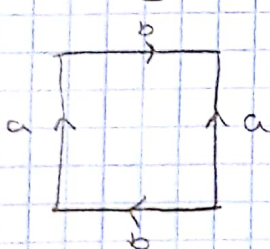
RP^2



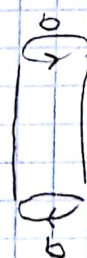
$T \# T$



$T \# T \# T$



Klein bottle



RP^2 , KB cannot be embedded in R^3 (Alexander, 1923)

Connected Sum:

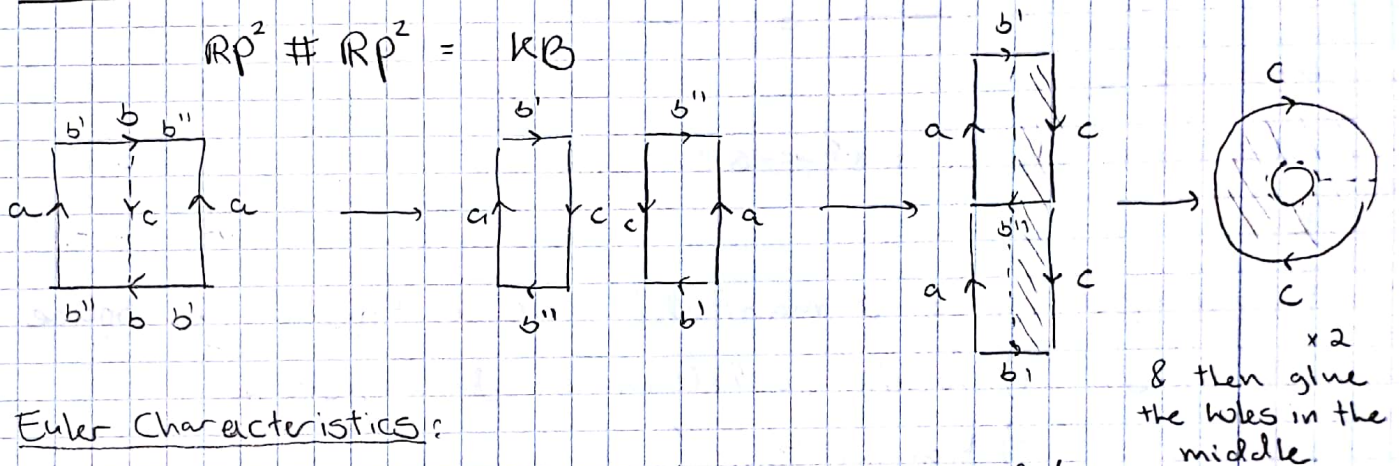
S_1, S_2 surfaces, the connected sum $S_1 \# S_2$ is obtained by removing a small piece from each & gluing along the boundary.

$$S^2 \# S = S$$

↑
any surface

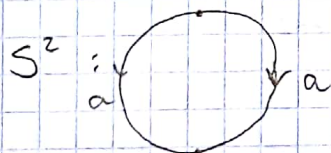
Claim:

$$\mathbb{R}P^2 \# \mathbb{R}P^2 = \mathbb{K}B$$

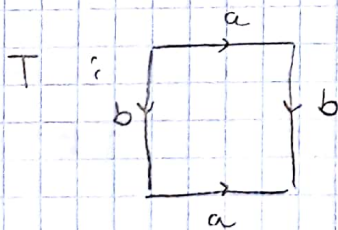


Euler Characteristics:

If a surface S is constructed as a simplicial / CW complex then $\chi(S) \stackrel{\text{def}}{=} \# \text{vertices} - \# \text{edges} + \# \text{faces}$.



$$\chi(S^2) = 2 - 1 + 1 = 2$$



$$\chi(T) = 1 - 2 + 1 = 0$$



$$\chi(\mathbb{R}P^2) = 1 - 1 + 1 = 1$$

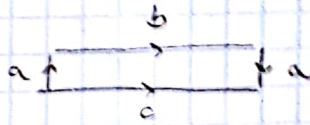
$$\begin{aligned} \chi(S_1 \# S_2) &= \chi(S_1) + \chi(S_2) - 2 \\ \chi(S_1 \setminus \text{Disc}) &= \chi(S_1) - 1 \\ \chi(S_2 \setminus \text{Disc}) &= \chi(S_2) - 1 \end{aligned}$$

$$\Rightarrow \chi(\underbrace{T \# \dots \# T}_{n \text{ times}}) = -2(n-1) = 2-2n$$

$$\chi(\underbrace{\mathbb{R}P^2 \# \dots \# \mathbb{R}P^2}_{n \text{ times}}) = n \cdot 1 - (n-1)2 = 2-n$$

Orientability:

Möbius band:



Def:

A surface S is not-orientable if it contains a Möbius band (embedded).

$\Leftrightarrow \exists$ a closed curve in the surface

