

7

3.7

מדידת מידה

מדידת מידה

$$A \cap B \subseteq \Omega$$

גורם

$$m^*(A \cup B) = m^*(A) + m^*(B) \quad \text{כאשר}$$

(*) נניח שיש לנו מדידת מידה m^* על Ω ונניח שיש לנו שני קבוצות $A, B \subseteq \Omega$ כאלו $A \cap B = \emptyset$

המשפט

נניח שיש לנו מדידת מידה m^* על Ω ונניח שיש לנו שני קבוצות $A, B \subseteq \Omega$ כאלו $A \cap B = \emptyset$

$$m^*(A \Delta A_\epsilon) \leq \epsilon \quad m^*(B \Delta B_\epsilon) \leq \epsilon \quad \text{עבור}$$

כל $\epsilon > 0$ אז $m^*(A \cap B_\epsilon) \leq \epsilon$ וכן $m^*(A_\epsilon \cap B) \leq \epsilon$

$$A_\epsilon \cap B_\epsilon \neq \emptyset \quad \text{עבור} \quad \emptyset = A \cap B \quad \text{עבור} \quad \text{מדידת מידה}$$

$$A_\epsilon \cap B_\epsilon \subseteq (A_\epsilon \setminus A) \cup (B_\epsilon \setminus B) \quad \text{כי} \quad A \cap B = \emptyset \quad \text{עבור} \quad \text{מדידת מידה}$$

$$m^*(A_\epsilon \cap B_\epsilon) \leq m^*(A_\epsilon \setminus A) + m^*(B_\epsilon \setminus B) < 2\epsilon$$

לכן $m^*(A_\epsilon \cap B_\epsilon) < 2\epsilon$

$$m(A_\epsilon \cup B_\epsilon) = m(A_\epsilon) + m(B_\epsilon) - \underbrace{m(A_\epsilon \cap B_\epsilon)}_{< 2\epsilon}$$

$$x \sim y \Leftrightarrow |x - y| < \epsilon \quad \text{כאשר} \quad \epsilon > 0 \quad x, y \in \mathbb{R}$$

$$m(A_\epsilon \cup B_\epsilon) \sim m(A_\epsilon) + m(B_\epsilon) \quad (*)$$

$$m^*((A \cup B) \Delta (A_\epsilon \cup B_\epsilon)) \leq m^*((A \Delta A_\epsilon) \cup (B \Delta B_\epsilon)) \leq 2\epsilon$$

$$(*) \quad m^*(A \cup B) \sim m^*(A_\epsilon \cup B_\epsilon) \quad \text{כאשר}$$

לכן

(המשפט 3.7)

$$|m^*(A) - m^*(B)| \leq m^*(A \Delta B) \quad \text{כאשר} \quad A, B \subseteq \Omega$$

המשפט

$$\begin{aligned} |m^*(A) - m^*(B)| &\leq m^*(A \Delta B) \\ m^*(A) &\leq m^*(B) + m^*(A \Delta B) \quad A \subseteq B \cup (A \Delta B) \\ m^*(B) &\leq m^*(A) + m^*(A \Delta B) \quad B \subseteq A \cup (A \Delta B) \end{aligned}$$

? וס' ר' נ' ס'c

$$m^*(B) \stackrel{(e)}{\sim} m^*(B_c) \quad m^*(A) \stackrel{(e)}{\sim} m^*(A_c)$$

$$m(A \cup B_c) \stackrel{(2e)}{\sim} m(A_c) + m^*(B_c) \quad (*_1)$$

$$m^*(A \cup B) \stackrel{(2e)}{\sim} m(A \cup B_c) \quad (*_2)$$

ס'c נ'c

$$m^*(A \cup B) \stackrel{(2e)}{\sim} m^*(A \cup B_c) \stackrel{(2e)}{\sim} m(A_c) + m(B_c) \stackrel{(2e)}{\sim} m(A) + m(B)$$

ס'c נ'c

$$m^*(A \cup B) \stackrel{(6e)}{\sim} m(A) + m(B)$$

$$\wedge N' p N' \quad \sim \text{ע'c} \text{ ו'c} \text{ ס'c} \quad A_c \cup B_c \quad -1 \quad ||'c -1$$

(*₂)

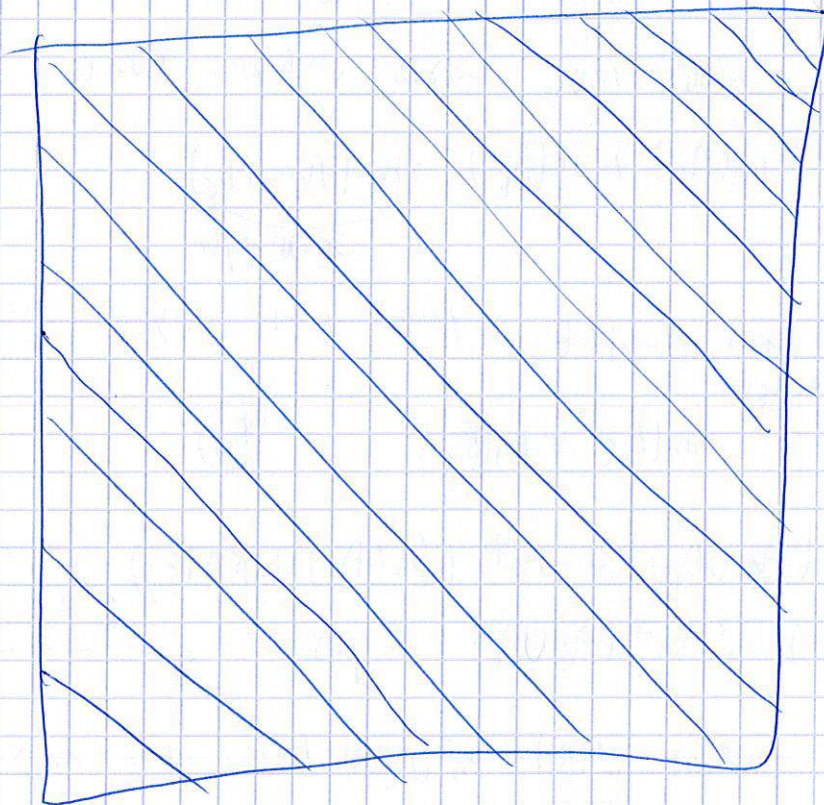
$$m^*((A \cup B) \Delta (A_c \cup B_c)) < 2\varepsilon$$

, נ'c נ'c

AUB

ס'c נ'c

$$\boxed{m(A \cup B) = m(A) + m(B)} \quad | \text{ס'c}$$



2

המשפט

3.1

המשפט

המשפט

$$m^*(A \cup B) = m^*(A) + m^*(B) \quad \text{if } A, B \subseteq \Omega \quad \text{given}$$

המשפט 3.1.1 (1) $m^*(A \cup B) \leq m^*(A) + m^*(B)$ (2) $m^*(A \cap B) \leq m^*(A) + m^*(B)$

$$m^*(B) \approx m^*(B_\epsilon)$$

$$m^*(A) \approx m^*(A_\epsilon)$$

המשפט

$$m^*(A_\epsilon \cap B_\epsilon) \leq m^*(A_\epsilon \setminus A) + m^*(B_\epsilon \setminus B) < 2\epsilon$$

$$m^*(A \cup B) \leq m^*(A_\epsilon \cup B_\epsilon) < 2\epsilon$$

$$m^*(A \cup B) \approx m^*(A_\epsilon \cup B_\epsilon)$$

$$m^*(A \cup B_\epsilon) \leq m^*(A_\epsilon \cup B_\epsilon) + 2\epsilon$$

$$|m^*(A) - m^*(B)| \leq m^*(A \Delta B)$$

$$A \subseteq B \cup A \setminus B \subseteq B \cup B \Delta A$$

$$A \subseteq A \cup B \setminus A$$

$$m^*(A) \leq m^*(B) + m^*(A \Delta B)$$

המשפט 3.1.1 (2) $m^*(A \cap B) \leq m^*(A) + m^*(B)$ (3) $m^*(A \cup B) \leq m^*(A) + m^*(B)$

$$m^*(A \cup B) = m(A) + m(B) - m(A \cap B) \quad (1)$$

$$m^*(A \cup B) = m(A \setminus B) + m(A \cap B) + m(B \setminus A) =$$

$$m(A \cup B) = m(A \setminus B) + m(A \cap B) + m(B \setminus A) = m(A \cup B)$$

$$A_1, \dots, A_n \quad (2)$$

$$m\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n m(A_i)$$

המשפט 3.1.2 (1) $m^*(A \cup B) \leq m^*(A) + m^*(B)$ (2) $m^*(A \cap B) \leq m^*(A) + m^*(B)$

משפט 1.1.1. $A_1, A_2, \dots$ μ - מדידת

משפט 1.1.2. $\sum_{i=1}^{\infty} \mu(A_i)$ μ - מדידת

משפט 1.1.3. $\{A_i\}_{i=1}^{\infty}$ μ - מדידת

$$\mu\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mu(A_i)$$

משפט 1.1.4. μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

הוכחה

משפט 1.1.5. μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

$$B_1 = A_1$$

$$B_n = A_n \setminus (A_1 \cup \dots \cup A_{n-1})$$

משפט 1.1.6. μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

$$\bigcup_{i=1}^{\infty} B_i = \bigcup_{i=1}^{\infty} A_i$$

$$\bigcup_{i=1}^n A_i = \bigcup_{i=1}^n B_i$$

משפט 1.1.7. μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

משפט 1.1.8. $A_1, A_2, \dots$ μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

$$\mu\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n \mu(A_i)$$

$$\sum_{i=1}^n \mu(A_i) \leq \mu(\Omega)$$

משפט 1.1.9. μ - מדידת $\mu(A) = \sum_{i=1}^{\infty} \mu(A_i)$

$$\sum_{i=1}^{\infty} \mu(A_i) < \infty$$

$$\sum_{i=1}^{\infty} \mu(A_i) \leq \mu(\Omega)$$

③ $\sum_{i=N_\epsilon}^{\infty} m(A_i) < \epsilon$ $\epsilon > 0$ N_ϵ N_ϵ $0 < \epsilon$ $\delta > 0$ $\delta > 0$

$m^* \left(\bigcup_{i=N_\epsilon}^{\infty} A_i \right) \leq \sum_{i=N_\epsilon}^{\infty} m(A_i) < \epsilon$

$\bigcup_{i=1}^{\infty} A_i$ $\delta > 0$ $\delta > 0$ $\delta > 0$

$A_i \in \mathcal{A}$

$m^*(A_i \Delta A_i \epsilon) \leq \frac{\epsilon}{N_\epsilon}$

$A_\epsilon = \bigcup_{i=1}^{N_\epsilon} A_i \epsilon$

$m^* \left(\bigcup_{i=1}^{\infty} A_i \right) \Delta A_\epsilon$

$m^* \left(\bigcup_{i=N_\epsilon}^{\infty} A_i \right) +$

$m^* \left(\left(\bigcup_{i=1}^{N_\epsilon} A_i \right) \Delta \bigcup_{i=1}^{\infty} A_i \right)$

$m^* \left(\bigcup_{i=N_\epsilon+1}^{\infty} A_i \right) < \epsilon$

$B = \bigcup_{i=1}^{N_\epsilon} A_i$

$m^*(B \Delta B_\epsilon) < \epsilon$ $B_\epsilon \subseteq \Omega$

$m^* \left(\left(\bigcup_{i=1}^{\infty} A_i \right) \Delta B_\epsilon \right) \leq m^* \left(\bigcup_{i=N_\epsilon+1}^{\infty} A_i \right) + m^*(B \Delta B_\epsilon) <$

$< \epsilon + \epsilon = 2\epsilon$

$\bigcup_{i=1}^{\infty} A_i$

① $\sum_{i=1}^{\infty} m(A_i) < \infty$

$m\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} m(A_i)$ ע וידוע (2) ע לפי (3)

$m^*\left(\bigcup_{i=1}^{\infty} A_i\right) \leq \sum_{i=1}^{\infty} m^*(A_i) < \infty$

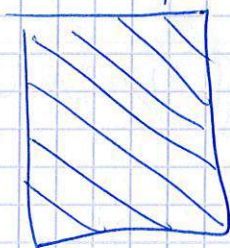
$\bigcup_{i=1}^{\infty} A_i \supseteq \bigcup_{i=1}^n A_i$

$m\left(\bigcup_{i=1}^{\infty} A_i\right) \geq m\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n m(A_i)$

לפי sup / inf

$m\left(\bigcup_{i=1}^{\infty} A_i\right) \geq \sum_{i=1}^n m(A_i)$

$m\left(\bigcup_{i=1}^{\infty} A_i\right) \geq \sum_{i=1}^{\infty} m(A_i)$



$m\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} m(A_i)$

לפי m ע לפי (3)

לפי $F \subseteq \mathcal{P}(\Omega)$ ע לפי (3)

לפי F ע לפי (3)

$\Omega \cap A = A^c \in F \leftarrow A \in F$ (2) ע לפי (3)

$\bigcup_{i=1}^{\infty} A_i \in F \leftarrow A_1, A_2, \dots \in F$ (3)

לפי F ע לפי (3)

לפי F ע לפי (3)

$\Omega \in \mathcal{P}(\Omega)$

לפי F ע לפי (3)

לפי F ע לפי (3)

לפי F ע לפי (3)

לפי F ע לפי (3)

✓ 100% (13)

(c)

[illegible]

שָׁמַר אֶת הַבְּרִית וְהָיָה לְעֵלֶם

1/ $\sqrt{2}$ A / $\rightarrow$ 1/12 (10) 3/11/5 1/55 ; 1/55 2/55 3/55 4/55 5/55 6/55 7/55 8/55 9/55 10/55 11/55 12/55 13/55 14/55 15/55 16/55 17/55 18/55 19/55 20/55 21/55 22/55 23/55 24/55 25/55 26/55 27/55 28/55 29/55 30/55 31/55 32/55 33/55 34/55 35/55 36/55 37/55 38/55 39/55 40/55 41/55 42/55 43/55 44/55 45/55 46/55 47/55 48/55 49/55 50/55 51/55 52/55 53/55 54/55 55/55 56/55 57/55 58/55 59/55 60/55 61/55 62/55 63/55 64/55 65/55 66/55 67/55 68/55 69/55 70/55 71/55 72/55 73/55 74/55 75/55 76/55 77/55 78/55 79/55 80/55 81/55 82/55 83/55 84/55 85/55 86/55 87/55 88/55 89/55 90/55 91/55 92/55 93/55 94/55 95/55 96/55 97/55 98/55 99/55 100/55 101/55 102/55 103/55 104/55 105/55 106/55 107/55 108/55 109/55 110/55 111/55 112/55 113/55 114/55 115/55 116/55 117/55 118/55 119/55 120/55 121/55 122/55 123/55 124/55 125/55 126/55 127/55 128/55 129/55 130/55 131/55 132/55 133/55 134/55 135/55 136/55 137/55 138/55 139/55 140/55 141/55 142/55 143/55 144/55 145/55 146/55 147/55 148/55 149/55 150/55 151/55 152/55 153/55 154/55 155/55 156/55 157/55 158/55 159/55 160/55 161/55 162/55 163/55 164/55 165/55 166/55 167/55 168/55 169/55 170/55 171/55 172/55 173/55 174/55 175/55 176/55 177/55 178/55 179/55 180/55 181/55 182/55 183/55 184/55 185/55 186/55 187/55 188/55 189/55 190/55 191/55 192/55 193/55 194/55 195/55 196/55 197/55 198/55 199/55 200/55 201/55 202/55 203/55 204/55 205/55 206/55 207/55 208/55 209/55 210/55 211/55 212/55 213/55 214/55 215/55 216/55 217/55 218/55 219/55 220/55 221/55 222/55 223/55 224/55 225/55 226/55 227/55 228/55 229/55 230/55 231/55 232/55 233/55 234/55 235/55 236/55 237/55 238/55 239/55 240/55 241/55 242/55 243/55 244/55 245/55 246/55 247/55 248/55 249/55 250/55 251/55 252/55 253/55 254/55 255/55 256/55 257/55 258/55 259/55 260/55 261/55 262/55 263/55 264/55 265/55 266/55 267/55 268/55 269/55 270/55 271/55 272/55 273/55 274/55 275/55 276/55 277/55 278/55 279/55 280/55 281/55 282/55 283/55 284/55 285/55 286/55 287/55 288/55 289/55 290/55 291/55 292/55 293/55 294/55 295/55 296/55 297/55 298/55 299/55 300/55 301/55 302/55 303/55 304/55 305/55 306/55 307/55 308/55 309/55 310/55 311/55 312/55 313/55 314/55 315/55 316/55 317/55 318/55 319/55 320/55 321/55 322/55 323/55 324/55 325/55 326/55 327/55 328/55 329/55 330/55 331/55 332/55 333/55 334/55 335/55 336/55 337/55 338/55 339/55 340/55 341/55 342/55 343/55 344/55 345/55 346/55 347/55 348/55 349/55 350/55 351/55 352/55 353/55 354/55 355/55 356/55 357/55 358/55 359/55 360/55 361/55 362/55 363/55 364/55 365/55 366/55 367/55 368/55 369/55 370/55 371/55 372/55 373/55 374/55 375/55 376/55 377/55 378/55 379/55 380/55 381/55 382/55 383/55 384/55 385/55 386/55 387/55 388/55 389/55 390/55 391/55 392/55 393/55 394/55 395/55 396/55 397/55 398/55 399/55 400/55 401/55 402/55 403/55 404/55 405/55 406/55 407/55 408/55 409/55 410/55 411/55 412/55 413/55 414/55 415/55 416/55 417/55 418/55 419/55 420/55 421/55 422/55 423/55 424/55 425/55 426/55 427/55 428/55 429/55 430/55 431/55 432/55 433/55 434/55 435/55 436/55 437/55 438/55 439/55 440/55 441/55 442/55 443/55 444/55 445/55 446/55 447/55 448/55 449/55 450/55 451/55 452/55 453/55 454/55 455/55 456/55 457/55 458/55 459/55 460/55 461/55 462/55 463/55 464/55 465/55 466/55 467/55 468/55 469/55 470/55 471/55 472/55 473/55 474/55 475/55 476/55 477/55 478/55 479/55 480/55 481/55 482/55 483/55 484/55 485/55 486/55 487/55 488/55 489/55 490/55 491/55 492/55 493/55 494/55 495/55 496/55 497/55 498/55 499/55 500/55 501/55 502/55 503/55 504/55 505/55 506/55 507/55 508/55 509/55 510/55 511/55 512/55 513/55 514/55 515/55 516/55 517/55 518/55 519/55 520/55 521/55 522/55 523/55 524/55 525/55 526/55 527/55 528/55 529/55 530/55 531/55 532/55 533/55 534/55 535/55 536/55 537/55 538/55 539/55 540/55 541/55 542/55 543/55 544/55 545/55 546/55 547/55 548/55 549/55 550/55 551/55 552/55 553/55 554/55 555/55 556/55 557/55 558/55 559/55 560/55 561/55 562/55 563/55 564/55 565/55 566/55 567/55 568/55 569/55 570/55 571/55 572/55 573/55 574/55 575/55 576/55 577/55 578/55 579/55 580/55 581/55 582/55 583/55 584/55 585/55 586/55 587/55 588/55 589/55 590/55 591/55 592/55 593/55 594/55 5

$\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$

הסדר הקבוע - Ω N γ δ

Ω '80975110 γ γ γ γ

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$\Omega_2 \supsetneq \Omega_1$ and $A \subseteq \Omega_2$ - 1

[illegible]
$$\text{für } p \geq 1 \quad m^*(A) =: m(A) \quad \text{für } A \in \mathcal{A} \quad - 2 \text{ S} \quad 5.7.02$$

גמולתו (1000) 1800

שני פונקציות - f ו- g מוגדרות על A ו- B כדלקמן:
 $f(x) = x^2 + 1$ ו- $g(x) = x + 1$

[illegible][illegible]
$$[0, 1]^2 \cup \mathbb{Q} \quad \text{Hausdorff} \quad (N)$$
$$\left\{ (x, y) \mid \begin{array}{l} x \geq 0 \\ 0 \leq y \leq \frac{1}{1+x^2} \end{array} \right\}$$
$$\sqrt{2} = [0, 1]^2 \quad 1/8, 3/8, 5/8, 7/8 \quad 1/4, 3/4$$

Ω se $\sim \frac{1}{\sqrt{\epsilon}}$ se $\sim \frac{1}{\sqrt{\epsilon}}$ se $\sim \frac{1}{\sqrt{\epsilon}}$ se $\sim \frac{1}{\sqrt{\epsilon}}$ se $\sim \frac{1}{\sqrt{\epsilon}}$

שני פונקציות f ו- g נקראות **אורתוגונליות** אם $\int_a^b f(x)g(x)dx = 0$

ה'תשנ"ב (1991) - פסח

$\{ \phi \} \delta$ ~~Alt~~ $\gamma \gamma \gamma$ $10 p \gamma$

$$\Omega_n \text{ is } 0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4$$

$$m(A) = \sum_{n=1}^{\infty} m(A \cap \Omega_n)$$

~~3. A $\rightarrow$ B~~

אם $\alpha \in \mathbb{R}$ אז $\alpha \in \mathbb{Q}$ או $\alpha \notin \mathbb{Q}$

1500, 1600, 1700, 1800, 1900, 2000, 2100, 2200, 2300, 2400, 2500, 2600, 2700, 2800, 2900, 3000, 3100, 3200, 3300, 3400, 3500, 3600, 3700, 3800, 3900, 4000, 4100, 4200, 4300, 4400, 4500, 4600, 4700, 4800, 4900, 5000, 5100, 5200, 5300, 5400, 5500, 5600, 5700, 5800, 5900, 6000, 6100, 6200, 6300, 6400, 6500, 6600, 6700, 6800, 6900, 7000, 7100, 7200, 7300, 7400, 7500, 7600, 7700, 7800, 7900, 8000, 8100, 8200, 8300, 8400, 8500, 8600, 8700, 8800, 8900, 9000, 9100, 9200, 9300, 9400, 9500, 9600, 9700, 9800, 9900, 10000

$\sqrt{N} \rightarrow \sqrt{3N} \rightarrow \sqrt{2N}$ Folge - Gren

$$A_1, A_2, \dots \subseteq \mathbb{R}^2 \quad \text{Nici} \quad \Rightarrow \exists \delta \in \mathbb{R} \quad \delta \quad \forall n, \forall N$$

$$m(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} m(A_i) \quad \text{NSI} \quad \text{NSI} \quad \text{NSI}$$

$$\bigcup_{h=0}^{\infty} \Omega_h = \mathbb{R}^2 \text{ so } \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$$

✓ 3-3N ϕ - 65cm ~ 100

$R^2 \approx 0.7534$ A_{adj} ~ 50

13777 6 108 0 3334 $\frac{1}{2}A$ 108

$$\gamma \beta' \beta n \quad \Omega_n \setminus A \quad \text{w} \quad \beta \delta \quad \gamma \beta' \beta n \quad A \cap \Omega_n$$

$$\Rightarrow 3'3'' \quad R^2_A = \bigcup_{h=0}^{\infty} L_h(A) \quad (18)$$

שנת ה'תשס"ה 5

$$\forall n \quad \left(\bigcup_{m=1}^{\infty} A_m \right) \cap \Omega_n = \bigcup_{m=1}^{\infty} (A_m \cap \Omega_n)$$

2. $m \in \mathbb{N}$

$$m\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{n=1}^{\infty} m\left(\bigcup_{i=1}^n A_i\right) \quad (\text{by } \sigma\text{-additivity}) \quad \sim \sim \sim \quad A_1, A_2, \dots \subseteq \mathbb{R}^d$$

⑧

$$m^*(A) = \sum_{n=1}^{\infty} m(A \cap \Omega_n) \quad A \subseteq \mathbb{R}^d \quad \text{and} \quad \sum$$

$$\mathbb{R}^2 = \cup \Omega_n, \text{ where } \Omega_n \text{ are disjoint sets}$$

$$: \leq \text{norm} - \text{norm}$$

$$A \subseteq \cup A \cap \Omega_n$$

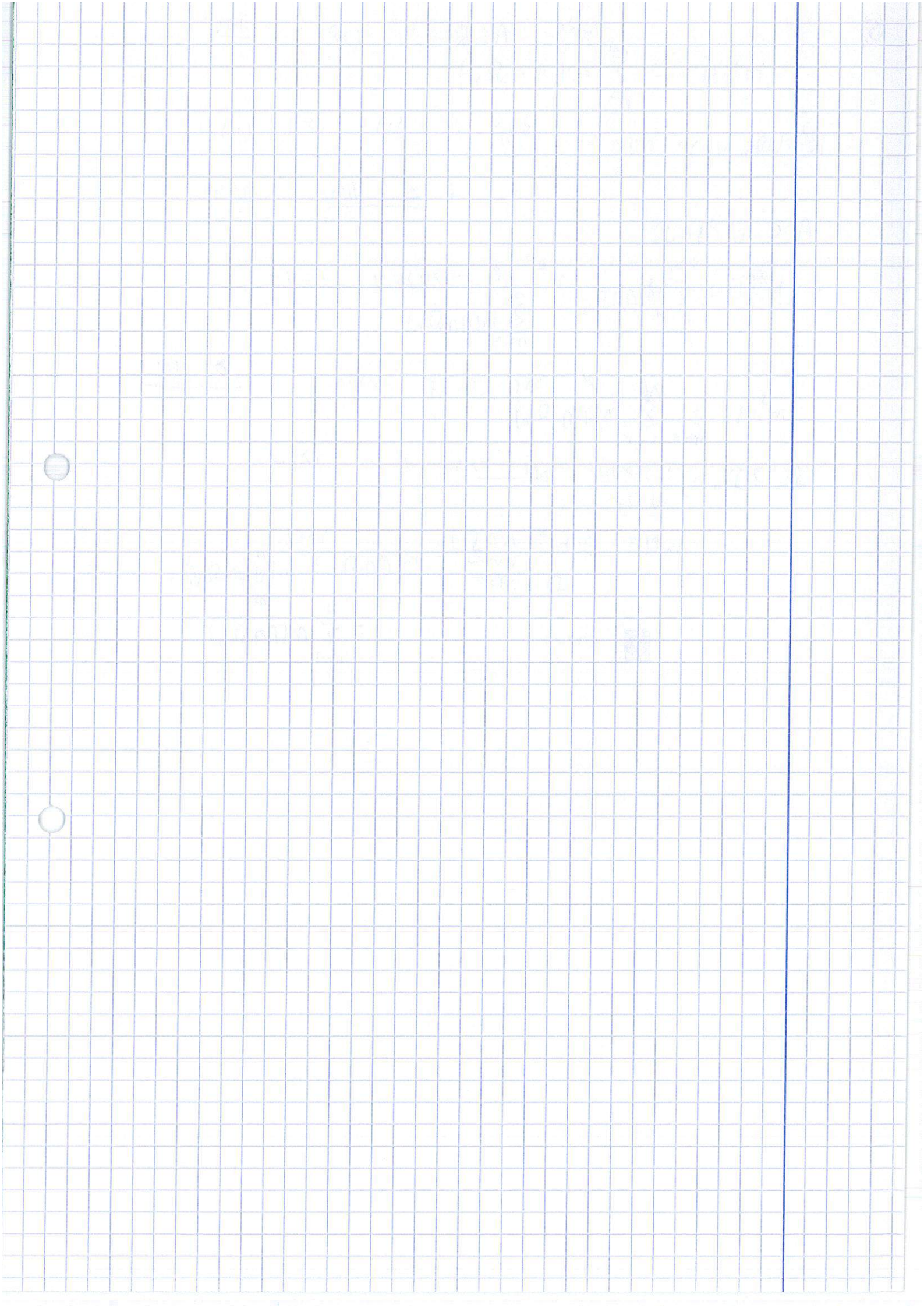
$$m^*(A) \leq \sum_{n=1}^{\infty} m^*(A \cap \Omega_n) = \sum_{n=1}^{\infty} m(A \cap \Omega_n)$$

$$m^*(A) \geq \sum_{n=1}^N m(A \cap \Omega_n) \quad \text{for any } N$$

$$\Omega \supseteq \Omega_1 \cup \dots \cup \Omega_N$$

$$m^*(A) \geq m^*(A \cap \Omega) = m(A \cap \Omega) \geq m(\cup_{n=1}^N A \cap \Omega_n) = \sum_{n=1}^N m(A \cap \Omega_n)$$

□ and



3.2 מידות הולדנר

Ω_n ~~הולדנר~~ $\mathbb{R}^2$ $A \subseteq \mathbb{R}^2$

$\Omega_n \supseteq \mathbb{R}^2$ $\forall n$ $A \cap \Omega_n$ $m(A) = m^*(A)$ $A \subseteq \mathbb{R}$

$\mathbb{R}^2$ $\mathbb{R}$ $\mathbb{R}^2$ $\mathbb{R}$

$A_i \subseteq \mathbb{R}^2$ $\mathbb{R}^2$ $\mathbb{R}$

$$m\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} m(A_n)$$

$A, B \subseteq \mathbb{R}^2$ $m(A \cup B) = m(A) + m(B) - m(A \cap B)$

$$m(A \cup B) = m(A) + m(B) - m(A \cap B)$$

$$m(A), m(B) < \infty$$

$$m(A \cap B) = \infty$$

$m(A \cup B) + m(A \cap B) = m(A) + m(B)$

$$m(A \cup B) + m(A \cap B) = m(A) + m(B)$$

$\mathbb{R}^2$ $\mathbb{R}$

$\mathbb{R}^2$ $\mathbb{R}$

$\mathbb{R}^2$ $\mathbb{R}$

$\mathbb{R}^2$ $\mathbb{R}$

$\mathbb{R}^2$ $\mathbb{R}$

$$A = \bigcup_{n=1}^{\infty} B\left(q_n, \frac{1}{2^n}\right)$$

$$\mathbb{Q}^2 = \{q_n\}_{n=1}^{\infty}$$

1.1.3

$$m(B(x, r)) \leq \pi r^2 \quad \text{for } A \subseteq \mathbb{R}^2 \text{ measurable}$$

$$\pi r^2$$

$$A \text{ is measurable}$$

$$m(A) \leq \sum_{i=1}^{\infty} m(B(q_i, \frac{1}{2^i})) \leq \pi \sum_{i=1}^{\infty} \frac{1}{2^i} < \infty$$

$$m(\mathbb{R}^2) = \infty \quad \text{not measurable}$$

$$m(A) = m^*(A) \quad \text{if } A \text{ is measurable}$$

$$m(A) \leq \sum m(A_i) \quad \text{if } A \subseteq \bigcup A_i$$

$$A \in \mathcal{M} = \bigcup_{n=1}^{\infty} B(q_n, \frac{\epsilon}{2^n})$$

$$A = \bigcap_{h=1}^{\infty} A_h$$

$$A \text{ is measurable}$$

$$A \text{ is measurable}$$

$$A \text{ is measurable}$$

$$A \text{ is measurable}$$

$$A \text{ is measurable}$$

$$A \subseteq \mathbb{R}^2$$

$$F, G \subseteq \mathbb{R}^2$$

$$F \subseteq A \subseteq G$$

$$m(G \setminus F) < \epsilon$$

$$m(G \setminus F) < \epsilon$$

$$25-3N \quad A \quad e \quad \delta^3 \Rightarrow$$

now ~~the~~ n 's'

[illegible]

Handwritten notes: $\Omega \geq 6\epsilon$, ϵp

$$m^{\frac{1}{2}} \left((G \cap \Omega) \Delta G_\varepsilon \right) < \varepsilon \quad ; \quad G \cap \Omega$$

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$$m^*(G \setminus A) \leq m(G \setminus F) < \epsilon$$

$$m^*((G \cap \Omega) \setminus (A \cap \Omega)) < \varepsilon$$

$$m^*((A \cap \Omega) \Delta G_e) \leq m^*((A \cap \Omega) \Delta (G \cap \Omega)) + m((G \cap \Omega) \Delta G_e)$$

[illegible]

$$\mu(G \setminus A) < \epsilon \quad \text{und} \quad A \subseteq G \quad \text{und}$$

Ans. $A \cap \Omega_n$ has n free μ

[illegible]

$$\sum_{i=1}^{\infty} m(p_{n,i}) \leq m(A_n \cap \Omega_n) + \frac{\varepsilon}{2^n} \quad A_n \cap \bigcup_{i=1}^{\infty} p_{n,i} \in \mathcal{E}$$

N_{gas} $N_{\text{gas}} n$ $\sim$ 6π f_{gas} γ_{rel} n^2 cm^{-3}

$$A_n \mathcal{Q}_n = \sum_{i=1}^{\infty} P_{n,i}^0$$

in $\mathbb{R}^n$ ~~also~~ $\mathbb{R}^n$ $A \in \mathbb{R}^n$

$$m\left(\bigcup_{i=1}^{\infty} P_{n,i}\right) \setminus (A \cap \Omega) \stackrel{\sqrt{1/2^{n-1} \epsilon}}{\downarrow} = m\left(\bigcup_{i=1}^{\infty} P_{n,i}\right) - m(A \cap \Omega_n) \leq \epsilon \cdot 2^{n-1} \cdot 1$$

$$\lim_{n \rightarrow \infty} \sum_{i=n}^{\infty} \ln(P_{n,i}) = \ln(A \cap \Omega_n)$$

$$m\left(\left(\bigcup_{i=1}^{\infty} P_{h,i}\right) \setminus (A \cap \Omega_h)\right) \leq \frac{\varepsilon}{2^h} \quad | \text{ד"ר} |$$

h פדס - סל $G = \bigcup_{i,h=1}^{\infty} P_{h,i}^0$ ד"ר

$$A \cap \Omega_h \subseteq G$$

מכאן של כל ה- $A \subseteq G$ ד"ר

$$\begin{aligned} m(G \setminus A) &\leq m\left(\left(\bigcup_{i,h=1}^{\infty} P_{h,i}^0\right) \setminus A\right) \leq \sum_{h=1}^{\infty} m\left(\left(\bigcup_{i=1}^{\infty} P_{h,i}^0\right) \setminus A\right) \leq \\ &\leq \sum_{h=1}^{\infty} m\left(\left(\bigcup_{i=1}^{\infty} P_{h,i}\right) \setminus (A \cap \Omega_h)\right) \leq \sum_{h=1}^{\infty} \frac{\varepsilon}{2^h} = \varepsilon \end{aligned} \quad | \text{ד"ר} |$$

אם A נכד ד"ר, נכד G נכד ד"ר $m(G \setminus A) < \varepsilon$

נכד ד"ר $m(G_n \setminus A^c) < \varepsilon$ $A^c \subseteq G_n$ ד"ר

נכד F $F = G_n^c$ נכד F נכד ד"ר $F \subseteq A$ $A^c \subseteq G_n$ ד"ר

$$m(A \setminus F) = m(A \Delta F) \leq m(A^c \Delta G_n) = m(G_n \setminus A^c) < \varepsilon \quad | \text{ד"ר} |$$

$m(A \setminus F) < \varepsilon$ נכד A נכד ד"ר נכד ד"ר

$$m(G \setminus F) = m(G \setminus A) + m(A \setminus F) < \varepsilon \quad | \text{ד"ר} |$$

$\uparrow$
נכד $G \setminus A$

נכד $\mathbb{R}^2 \supseteq A$ ד"ר נכד

$$m(A) = \inf \left\{ m(G) \mid \begin{array}{l} \text{נכד } G \\ A \subseteq G \end{array} \right\}$$

$$m(A) = \sup \left\{ m(F) \mid \begin{array}{l} \text{נכד } F \\ F \subseteq A \end{array} \right\}$$

נכד $F \subseteq G$ נכד $F \subseteq A$ $\varepsilon < \varepsilon$ ד"ר

$$m(A \setminus F) < \varepsilon \quad m(G \setminus A) < \varepsilon$$

$$m(G) \leq m(A) + m(G \setminus A) \stackrel{m(A) < \infty}{\leq} m(A) + \epsilon$$

$m(G)$ איז א נומער וואס איז גרעסער ווי $m(A)$ און נעמט איר אן צו געבן א שטארקע אומגלעכקייט

און $m(A) < \delta$ איז א קליין נומער

און $m(A) = \infty$ איז א נומער וואס איז גרעסער ווי δ

און $m(A) < \delta$ איז א קליין נומער

G_δ איז א קליין נומער $A \subseteq \mathbb{R}^2$ איז א קליין נומער

און $m(G_\delta) < \delta$ איז א קליין נומער

און $m(G_\delta) < \delta$ איז א קליין נומער

G_δ איז א קליין נומער F_δ איז א קליין נומער

F_δ איז א קליין נומער

G_δ איז א קליין נומער $A \in \mathcal{M}$ איז א קליין נומער

F_δ איז א קליין נומער $A \in \mathcal{M}$ איז א קליין נומער

$$\begin{array}{ccc} F \subseteq A \subseteq G & \mathcal{M}' & A \text{ איז א קליין נומער} \\ \downarrow & & \downarrow \\ F_\delta & & G_\delta \end{array}$$

$$m(G \setminus F) = 0 \quad \text{ע.ג.}$$

$$\epsilon = \frac{1}{h} \quad \text{און } h > 0 \quad \text{און } h < \infty \quad \text{און } h > 0$$

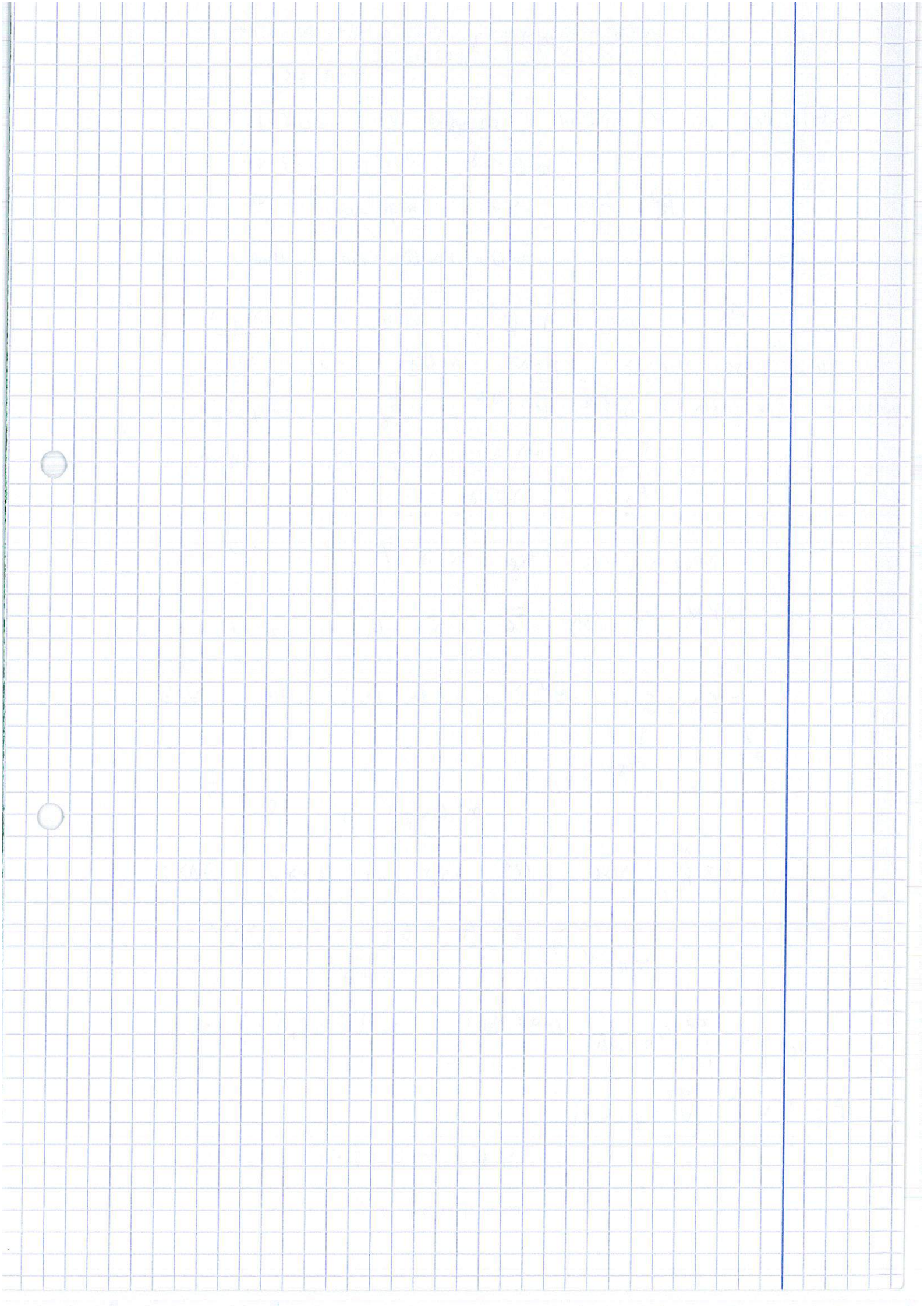
$$F_h \subseteq A \subseteq G_h \quad \text{און } h > 0$$

$$G_\delta \text{ איז } G = \bigcap_{h=1}^{\infty} G_h \supseteq A \quad \text{און } m(G_h \setminus F_h) < \frac{1}{h} \quad \text{און}$$

$$F_\delta \text{ איז } F = \bigcup_{h=1}^{\infty} F_h \subseteq A$$

$$\forall \epsilon \quad m(G \setminus F) < \epsilon \quad \text{און}$$

$$m(G \setminus F) = 0 \quad \text{און}$$



27/10/13

3

8/12/13

המשפט

המשפט

$A \subseteq \mathbb{R}^n$

המשפט

$\epsilon > 0$ E $\sim \text{בממד } n$

$\exists$ $\delta > 0$ $0 < \delta$

$\delta > 0$ $\sim$

$$m^*(A \Delta E) < \epsilon$$

$$m^*(A \cup B) = m(A) + m(B)$$

ש"כ

$\sim$

$\sim$

$A, B \subseteq \mathbb{R}^n$

נ"ל

נ"ל

I

$\sim$

$A \subseteq \mathbb{R}^n$

$\sim$

$\sim$

$\sim$

המשפט

$$0 \leq d_A(I) = \frac{m(A \cap I)}{m(I)} \leq 1$$

$$m(A) > 0$$

$\sim$

$A \subseteq \mathbb{R}^n$

$\sim$

המשפט

I $\sim$

$\sim$

$\sim$

$\lambda < 1$

$\sim$

$$d_A(I) > \lambda$$

$\sim$

$$0 < \epsilon$$

$\sim$

$$m(E \Delta A) < \epsilon \cdot m(A) \quad \epsilon > 0 \quad \sim \text{בממד } n \quad E \quad \sim$$

$$m(A) = m(A \setminus E) + m(A \cap E) \leq m(A \Delta E) + m(A \cap E) < \epsilon \cdot m(A) + m(A \cap E)$$

$\sim$

$$(*) \quad m(A) < \frac{m(A \cap E)}{1 - \epsilon}$$

$$m(E) = m(E \cap A) + m(E \setminus A) < m(A \cap E) + \epsilon \cdot m(A)$$

$$m(E) < \frac{m(A \cap E)}{1 - \epsilon} + \epsilon \cdot m(A) < m(A \cap E) + \epsilon \cdot \frac{m(A \cap E)}{1 - \epsilon} =$$

$$= m(A \cap E) \left(\frac{1}{1 - \epsilon} \right) \quad (*) \quad \Rightarrow (1 - \epsilon)m(E) < m(A \cap E)$$

$$E = \bigcup_{i=1}^{\infty} I_i$$

$\sim$

$\sim$

$\sim$

$\sim$

$$(1 - \epsilon) m(E) = (1 - \epsilon) \cdot \sum_{i=1}^{\infty} m(I_i)$$

$$m(A \cap E) = \sum_{j=1}^{\infty} m(A \cap I_j) = \sum_{j=1}^{\infty} \frac{m(A \cap I_j)}{m(I_j)} m(I_j)$$

$$\sum_{j=1}^n \frac{m(A \cap I_j)}{m(I_j)} m(I_j) = m(A \cap E) > (1-\epsilon)m(E) = \sum_{j=1}^n (1-\epsilon)m(I_j)$$

$$m(A \cap E) > (1-\epsilon)m(E)$$

$$\frac{m(A \cap I_j)}{m(I_j)} m(I_j) > (1-\epsilon)m(I_j)$$

$$\frac{m(A \cap I_j)}{m(I_j)} > 1-\epsilon$$

$$1-\epsilon = \epsilon$$

אם ϵ קטן מספיק

אז $m(A \cap E) > (1-\epsilon)m(E)$ וזה סותר ל- $m(A \cap E) < \epsilon m(E)$

$$E_n \subseteq \mathbb{R}^2 \quad \{E_n\}_{n=1}^{\infty} \quad \text{סדרה}$$

$$E = \bigcap_{n=1}^{\infty} E_n \quad E_1 \supseteq E_2 \supseteq E_3 \supseteq \dots \quad (i)$$

$$E = \bigcup_{n=1}^{\infty} E_n \quad E_n \supseteq E \quad E_1 \subseteq E_2 \subseteq E_3 \dots \quad (ii)$$

$$E_n \nearrow E$$

$$\mathbb{R}^2 \supset E \supset E_n \supset E \quad m(E_n) \rightarrow m(E)$$

$$m(E_n) \rightarrow m(E)$$

$$m(E_n) < \infty \quad E_n \nearrow E \quad m(E_n) \rightarrow m(E)$$

$$m(E_n) \rightarrow m(E) \quad E \text{ סגור}$$

$$E = \bigcup_{n=1}^{\infty} E_n \quad m(E) = \lim_{n \rightarrow \infty} m(E_n)$$

$$G_k = E_k \setminus E_{k-1}$$

$$E = \bigcup_{n=1}^{\infty} G_n$$

$$\{G_n\}_{n=1}^{\infty}$$

$$G_1 = E_1$$

$$m(E) = m(\bigcup_{n=1}^{\infty} G_n) = \sum_{n=1}^{\infty} m(G_n)$$

$$m(E) = \lim_{N \rightarrow \infty} \sum_{n=1}^N m(G_n) = \lim_{N \rightarrow \infty} m(E_N) = \lim_{N \rightarrow \infty} m(E_N)$$

($\mathbb{R}^n / \mathbb{R}^n$) $\mathbb{R}^n$ $\mathbb{R}^n$ $\mathbb{R}^n$

$$\chi_A(x) = \chi_A(x) = \chi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

$$E \subseteq \mathbb{R}^n \quad \mathbb{R}^n \supset \{E_n\}_{n=1}^{\infty} \quad \text{משפחה פתוחה}$$

$$E \supset \bigcup_{n=1}^{\infty} E_n \quad \chi_E = \bigvee_{n=1}^{\infty} \chi_{E_n}$$

$$E_n \rightarrow E \quad \mathbb{R}^n \supset \{E_n\}_{n=1}^{\infty} \quad \text{משפחה פתוחה}$$

$$m(E_n) \rightarrow m(E) \quad \text{אם } E = \bigcup_{n=1}^{\infty} E_n \text{ ו- } m(E) < \infty$$

משפחה פתוחה $\mathbb{R}^n$ $\mathbb{R}^n$ $\mathbb{R}^n$ $\mathbb{R}^n$

$$(E_n = [n, n+1]) \quad E_n \rightarrow \emptyset$$

$$\chi_E(x) = \liminf \chi_{E_n}(x) = \limsup \chi_{E_n}(x)$$

$$\sup_{k \geq n} \inf_{h \geq k} \chi_{E_h}(x) \quad \inf_{k \geq n} \sup_{h \geq k} \chi_{E_h}(x)$$

$$\sup_{h \geq k} \chi_{E_h}(x) = \chi_{\bigcup_{h \geq k} E_h}(x)$$

$$\inf_{h \geq k} \chi_{E_h}(x) = \chi_{\bigcap_{h \geq k} E_h}(x)$$

$$E = \bigcap_{k=1}^{\infty} \bigcup_{h \geq k} E_h = \bigcap_{k=1}^{\infty} \bigcup_{h \geq k} E_h$$

משפחה פתוחה $\mathbb{R}^n$ $\mathbb{R}^n$ $\mathbb{R}^n$ $\mathbb{R}^n$

$$B_k \supseteq E_k \quad A_k \subseteq E_k \quad \text{משפחה פתוחה}$$

$$m(E) = m\left(\bigcap_{k=1}^{\infty} B_k\right) = \lim_{k \rightarrow \infty} m(B_k) \geq \lim_{k \rightarrow \infty} m(A_k)$$

