**Graph Theory: Homework Assignment Number 2**

Due: Wednesday Dec. 24.

1. Prove that in a connected graph $G$ every two paths of maximum length share a vertex.

2. What is the minimum possible number of edges in a 6-connected graph on 2000 vertices ? Give a construction and show it is best possible.

3. Let $k \geq 2$. Prove that every $k$-connected graph on at least $2k$ vertices contains a cycle of length at least $2k$.

4. Prove that every graph with $n > 1$ vertices and at least $2n$ edges contains a cycle of length at most $2 \log_2 n$.
   Hint: First show that each such graph contains a subgraph with minimum degree at least 3.

5. Let $G = (V, E)$ be a simple graph with 101 vertices and minimum degree 51 and let $v \in V$ be a vertex. Prove that $G$ contains a simple cycle of length precisely 27, which contains $v$.

6. Let $G = (V, E)$ be the graph whose vertices are the $4n$ squares of a 4 by $n$ "chess-board", where two vertices are adjacent if and only if a knight can jump between the corresponding squares. (Therefore, the set of vertices is $\{(i, j) : \ 1 \leq i \leq 4, \ 1 \leq j \leq n\}$, and $(i, j)$ is adjacent by an edge to $(i', j')$ if and only if either $|i - i'| = 1$ and $|j - j'| = 2$,
   or
   $|i - i'| = 2$ and $|j - j'| = 1$.
   Does $G$ contain a Hamilton cycle? Prove your claim!