

25/03/2012

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11/3/2

$$f'(z_0) = \lim_{h \rightarrow 0} \frac{f(z_0+h) - f(z_0)}{h}$$

$$f = u + iv$$

11/3/2

$$\frac{\partial u}{\partial x}(z_0) = \frac{\partial v}{\partial y}(z_0)$$

$$\frac{\partial u}{\partial y}(z_0) = -\frac{\partial v}{\partial x}(z_0)$$

11/3/2 11/3/2 $u, v \in C^1(G)$ 11/3/2
 $A(G) \rightarrow f = u + iv$ 11/3/2

$$u + iv \in A(G), \quad u, v \in C^2(G) \quad 11/3/2$$

$$u, v \in \text{Har}(G) \quad 11/3/2$$

$$\Delta u = \Delta v \equiv 0 \quad 11/3/2$$

$$\Delta u = u_{xx} + u_{yy} \quad 11/3/2$$

$$u(x,y) = xy$$

$$\begin{cases} \frac{\partial v}{\partial y}(x,y) = y \\ \frac{\partial v}{\partial x}(x,y) = x \end{cases}$$

$$v(0,0) = 0$$

$$\varphi(x) = v(x,0)$$

$$\varphi'(x) = v_x(x,0) = x \Rightarrow \varphi(a) = v(a,0) = \int_0^a \varphi'(x) dx = \frac{1}{2}a^2 + C$$

$$0 = \varphi(0) = v(0,0) = 0 + C$$

$$\Rightarrow C = 0$$

$$v(a,0) = \frac{1}{2}a^2$$

$$\varphi(y) = v(a,y)$$

$$v(a,b) = \int_0^b \varphi'(y) dy + v(a,0) =$$

$$= \int_0^b y dy + \left(-\frac{1}{2}a^2\right) = \frac{b^2}{2} - \frac{a^2}{2}$$

$$\Rightarrow v(x,y) = \frac{y^2}{2} - \frac{x^2}{2}$$

$$f(x,y) = xy + i\left(\frac{y^2}{2} - \frac{x^2}{2}\right) =$$

$$= \frac{1}{2i} (x^2 - y^2 + 2ix) = \frac{1}{2i} (x+iy)^2 = \frac{1}{2i} z^2$$

המשקל הנכונים :

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}, \quad u(x, y)$$

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$$\psi(x) = v(x, 0)$$

$$\psi(b) - \psi(0) = \int_0^b \psi'(x) dx = \int_0^b -u_y(x, 0) dx$$

$$\psi(y) = v(a, y)$$

$$\psi(b) - \psi(0) = \int_0^b \psi'(y) dy = \int_0^b u_x(a, y) dy$$

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$$v(a, b) = \psi(b) = \int_0^b u_x(a, y) dy + \psi(0) =$$

$$= \int_0^b u_x(a, y) dy + \int_0^a u_y(x, 0) dx$$

$$v(x, y) = \int_0^y u_x(x, t) dt - \int_0^x u_y(s, 0) ds$$

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$$\frac{\partial}{\partial y} v(x, y) = u_x(x, y)$$

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$$\frac{\partial}{\partial x} v(x, y) = -u_y(x, y)$$

$$V_1(x, y) = \int_0^y u_x(0, t) dt + \int_0^x -u_y(s, y) ds$$

$$V = V_1$$

$$v(x, y) - v_1(x, y) = \underbrace{\int_0^y u_x(x, t) - u_x(0, t) dt}_{I_1} + \underbrace{\int_0^x u_y(s, y) - u_y(s, 0) ds}_{I_2}$$

$$\varphi(t) = u_y(s, t)$$

$$\begin{aligned} \varphi(y) - \varphi(0) &= \\ &= \int_0^y \varphi'(t) dt \end{aligned}$$

$$\varphi'(t) = u_{yy}(s, t)$$

$$I_2 = \int_0^x \int_0^y u_{yy}(s, t) dt ds$$

$$\varphi(s) = u_x(s, t)$$

$$\varphi'(s) = u_{xx}(s, t)$$

$$I_1 = \int_0^y \int_0^x u_{xx}(s, t) ds dt$$

$$I_2 = \int_0^x \int_0^t u_{yy}(s, t) ds dt$$

$$I_1 = \int_0^x \int_0^t u_{xx}(s, t) ds dt$$

$$V - V_1 = I_1 + I_2 = \int_0^x \int_0^t u_{yy}(s, t) + u_{xx}(s, t) ds dt =$$

$$= \int_0^x \int_0^t \Delta u ds dt = \int_0^x \int_0^t 0 ds dt = 0$$

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