

ש"ס. ש"ס.

$$\arg(z) = \theta = \arctan\left(\frac{y}{x}\right) \text{ mod } (2\pi)$$

$$Z_1 \sim \Gamma_1, \theta_1$$

$$z_1, z_2 \sim r_1 \cdot r_2, \quad \theta_1 + \theta_2 \pmod{2\pi}$$

:- 2 PCN

$z^n = a$ ($a \in \mathbb{C}$)

$|z| = r^n, \arg(z) = \theta$ 'st

$$K = 0, 1, 2, \dots, N-1$$

$$: a=1 \quad \text{728}$$

$$z^n = 1$$

$$|z| = 1$$

$$\arg z = \frac{2k\pi}{n} \quad k \in 0, 1, \dots, n-1$$

$$: \text{סדר} - \text{דיונים} \quad \text{2413}$$

$$: \text{2413}$$

$$S = \sin \theta + \sin 2\theta + \dots + \sin n\theta$$

$$: \text{2413}$$

$$\tilde{S} = 1 + \cos \theta + \cos 2\theta + \dots + \cos n\theta$$

$$\tilde{S} + iS = 1 + z + z^2 + \dots + z^n = \frac{z^{n+1} - 1}{z - 1}$$

$$z = \text{cis } \theta \quad \text{728}$$

$$S = \text{Im} \left(\frac{z^{n+1} - 1}{z - 1} \right) \quad \text{728}$$

$$: \text{728}$$

$$w = \text{cis } \frac{\theta}{2}$$

$$z = w^2 \quad \text{728}$$

$$w \cdot \bar{w} = 1 \quad \text{728}$$

$$: \text{728} \quad \frac{1}{w} = \bar{w} \quad \text{728}$$

$$\begin{aligned} z - 1 &= w^2 - 1 = w(w - \frac{1}{w}) = \cancel{w} \cdot \cancel{w} \\ &= w \cdot (w - \bar{w}) = w \cdot (2i \sin \frac{\theta}{2}) \end{aligned}$$

$$z^{n+1} - 1 = w^{n+1} \cdot 2i \sin \frac{(n+1)\theta}{2} \quad \text{728}$$

$$S = \text{Im} \left(\frac{w^{n+1}}{w} \cdot \frac{2i \sin \frac{(n+1)\theta}{2}}{2i \sin \frac{\theta}{2}} \right) = \text{Im} \left(w^n \cdot \frac{\sin \frac{(n+1)\theta}{2}}{\sin \frac{\theta}{2}} \right)$$

$$S = \operatorname{Im} \left(\cos \frac{n\theta}{2} + i \sin \frac{n\theta}{2} \right) \frac{\sin \frac{(n+1)\theta}{2}}{\sin \frac{\theta}{2}} =$$

$$= \frac{\sin \frac{n\theta}{2} \cdot \sin \frac{(n+1)\theta}{2}}{\sin \frac{\theta}{2}}$$

$$2 |A|^2 = |A|^2$$

(positive) for all θ

$$\left| \sum_{j=1}^n z_j \cdot w_j \right| \leq \sqrt{\sum_{j=1}^n |z_j|^2} \cdot \sqrt{\sum_{j=1}^n |w_j|^2}$$

$$\sum |w_j|^2 = 1, \sum |z_j|^2 = 1$$

dim

$$\left| \sum z_j w_j \right| \leq \sum |z_j| |w_j|$$

$$ab \leq \frac{a^2 + b^2}{2}$$

by AM-GM inequality

$$\leq \sum \frac{|z_j|^2 + |w_j|^2}{2} = \frac{1}{2} \sum |z_j|^2 + \frac{1}{2} \sum |w_j|^2 = \frac{1}{2} + \frac{1}{2} = 1$$

QED

(2) מוקד :

הערה : 0 איננו מקרה (1)

$$A = \sum |z_j|^2 > 0$$

$$B = \sum |w_j|^2 > 0$$

$$u_j = \frac{z_j}{\sqrt{A}}$$

$$v_j = \frac{w_j}{\sqrt{B}}$$

(3) :

355 (11)

$$\sum |u_j|^2 = \frac{1}{A} \sum |z_j|^2 = 1$$

$$\sum |v_j|^2 = \frac{1}{B} \sum |w_j|^2 = 1$$

(4)

הערה : (1) מוקד :

$$|\sum u_j v_j| \leq 1$$

$$|\frac{1}{\sqrt{A}} \cdot \frac{1}{\sqrt{B}} \sum z_j w_j| \leq 1$$

52

$$|\sum z_j w_j| \leq \sqrt{AB} = \sqrt{\sum |z_j|^2} \sqrt{\sum |w_j|^2}$$

See

732) מרחק בין שני נקודות

$$\rho(z_1, z_2) \cdot |z_1 - z_2| = |z_2 - z_1|$$

$$z_1 = z_2 \Rightarrow \rho(z_1, z_2) = 0, \quad \rho \geq 0, \quad \rho(z, w) = \rho(w, z)$$

$$\rho(z_1, z_2) \leq \rho(z_1, z_3) + \rho(z_3, z_2) \quad \text{למשל} \quad \text{etc.}$$

732) מרחק

$$\rho(z_n, w) \xrightarrow{n \rightarrow \infty} 0 \quad \text{אם} \quad z_n \rightarrow w$$

732) מרחק

$$r > 0 \quad D_r(w) = \{z \in \mathbb{C} \mid \rho(z, w) < r\}$$

732) מרחק

$$A \subseteq \mathbb{C} \quad \text{מקום}$$

$$A \subseteq \mathbb{C} \quad \text{ז} \in \text{Int}(A) \quad \text{ז} \in A \quad (1)$$

$$D_r(z) \subset A \quad \text{ע} \quad \rho \quad r > 0 \quad \text{מקום}$$

$$D_r(z) \cap A = \emptyset \quad \text{ע} \quad \rho \quad z \in \text{Ext}(A) \quad \text{ז} \in A \quad (2)$$

$$\text{Ext}(A) = \text{Int}(A^c)$$

$$A \subseteq \mathbb{C} \quad z \in \text{Fr}(A) \quad (\partial A) \quad (3)$$

$$D_r(z) \cap A^c \neq \emptyset \quad \text{ע} \quad D_r(z) \cap A \neq \emptyset$$

$$A = \text{Int} A \quad \text{מקום} \quad A \text{ פתוח} \quad (4)$$

$$\text{Fr}(A) \subset A \quad \text{מקום} \quad A$$

$$A^c \quad \text{מקום} \quad A$$

$$z_1 \rightarrow w \in \mathbb{C} \quad \text{ע} \quad \rho \quad z_1 \quad \text{מקום} \quad A$$

תוצאה:

$$\begin{array}{llll} \text{מקור} & \text{פולי} & V_{\alpha} & (1) \\ \text{מקור} & \cup & V_{\alpha} & \text{ש} \end{array}$$

$$\text{מקור} \quad \text{פולי} \quad \bigcap_{i=1}^n V_{\alpha} \quad (2)$$

$$\text{מקור} \quad \text{פולי} \quad F_{\alpha} \quad (3)$$

$$\text{מקור} \quad \bigcap F_{\alpha} \quad \text{ש}$$

$$\text{מקור} \quad \bigcup_{i=1}^n F_{\alpha} \quad (4)$$

$$\text{מקור} \quad \text{פולי} \quad \phi, \mathbb{C}$$