Complex function theory - recitation

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1 Elemtary funcitons

1. $$Hol\left(\mathbb{C}\right) \ni e^{z} := e^{x} \cdot e^{iy} := e^{x} \cdot \sin y + e^{x} \cdot \cos y$$

2. $$Hol\left(\mathbb{C}\right) \ni \sin z = \frac{e^{iz} - e^{-iz}}{zi}$$

3. $$Hol\left(\mathbb{C}\right) = \frac{e^{iz} + e^{-iz}}{2}$$

4. ...

e^{z} is $2\pi i$-periodic, $\cos z, \sin z$ are 2π-periodic. On the other hand, $\cos z / \sin z$ is not periodic.
We define $\text{Log}z$ as te set of all the solutions to the equation $e^{w} = z$ e.g. $w \in \text{Log}z$ iff $e^{w} = z$. $\text{Log}z = \overbrace{\ln|z|}^{\in\mathbb{R}} + i \cdot \text{Arg}z$ where $\text{Arg}z$ denotes the set of all the arguments. A branch of the logarithm is $\log z = \ln|z| + i \cdot \alpha(z)$ where α represents a one-to-one choice of the argument. For example, $\log z = \ln|z| + i \arg z, \arg \in (-\pi, \pi]$. $\log z$ is analytic in $\mathbb{C}\backslash(-\infty, 0]$. If we choose Arg in the section $[0, 2\pi)$ then log is analytic in $\mathbb{C}\backslash[0, \infty)$.!!!!

Example 1. $i^{\sqrt{3}}$ find the set.

$$i^{\sqrt{3}} := e^{\sqrt{3}\text{Log}i} = e^{\sqrt{3} + \ln|i| + i\text{Arg}i} = e^{i\sqrt{3}\left(\pi/2 + 2\pi k\right)} = e^{i\sqrt{3}\frac{\pi}{2} + \sqrt{3}2\pi k} \Rightarrow i^{\sqrt{3}} = \left\{e^{i\sqrt{3}\frac{\pi}{2} + \sqrt{3}2\pi k} : k \in \mathbb{Z}\right\}$$

Problem 1. The problem here is that the lesson is mostly visual, we will upload a written version. In general, it is similar to last recitation where instead of using möbius transformations we found biholomorphic maps.