

Lex δ Σ Γ Δ

$$G_{ex} = \{ \{s_0, s_1, \dots, s_n\}, \Sigma, R, s_0 \}$$

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R:

$$s_0 \rightarrow 1 s_1 \mid 2 s_2 \mid =$$

$$s_1 \rightarrow 1 s_2 \mid 2 s_3$$

$$s_2 \rightarrow 1 s_3 \mid 2 s_4$$

$$s_3 \rightarrow 1 s_4 \mid 2 s_0 s$$

$$s_4 \rightarrow 1 s_0 s \mid 2 s_1 s$$

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$$PDA \quad P = (Q_P, \Sigma, \Gamma, \delta_P, q_{P0}, F_P)$$

$$DFA \quad A = (Q_A, \Sigma, \delta_A, q_{A0}, F_A)$$

$$P' = (Q_P \times Q_A, \Sigma, \Gamma, \delta_{P'}, (q_{P0}, q_{A0}), F_P \times F_A)$$

$$\delta_{P'}((q_P, q_A), \sigma, a) = \left\{ (r, s), b \mid \begin{array}{l} (r, b) \in \delta_P(q_P, \sigma, a) \\ s = \delta_A(q_A, b) \end{array} \right\}$$

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$$w = uvxyz \quad \text{...} \quad \forall w \in L, |w| \geq p \quad p \geq 1$$

$$i \quad \text{...} \quad |vxy| \leq p \quad |vy| \geq 1 \quad p \geq 1$$

$$uv^i x y^i z \in L$$

$$L = \{a^i b^j c^k \mid j = \max\{i, k\}\}$$

האם L היא שפה רגולרית?

$$w = a^p b^p c^p$$

נבחר p כרצוננו

$$w = uvxyz$$

$$|vy| \geq 1 \quad |vxy| \leq p$$

נניח $|v| \geq 1$

$$b \in vxy$$

$$w' = vxz$$

אם $|v| \geq 1$ אז $|v| \leq p$

$$0 \leq i$$

אם $|v| \geq 1$ אז $|v| \leq p$ ונבחר p כרצוננו

$$w' \notin L$$

$$vxy = a^p b^p c^p \quad \text{אם } |v| \geq 1 \text{ אז } |v| \leq p$$

$$c \text{ אינו שייך ל-} v$$

Example 4

Show that the following language is not a CFG:

$$L = \{a^i b^j c^k \mid j = \max(i, k)\}$$

Assume that L is a CFL and let p be the pumping length.

We will choose $w = a^p b^p c^p$. Clearly, $|w| > p$

Let $w = uvxyz$ such that $|vy| \geq 1$ and $|vxy| \leq p$
As $|vxy| \leq p$ it contains at most two letters

If vy contains the letter b , then let us look at $w' = uxz$ (we pump by 0)
Clearly the number of b 's was reduced but either the number of a 's or c 's was not reduced thus $j \neq \max(i, k)$

If vy does not contain the letter b , then let us look at $w' = uv^2xy^2z$

Clearly the number of b 's has not changed but either the number of a 's or c 's was increased thus $j \neq \max(i, k)$

הוכחה 7

$$L = \{w \bar{w} \mid w \in \Sigma^*\}$$

$$\Sigma = \{0, 1\}$$

$$w \in \bar{A}$$

הוכחה 7

$$w = \underbrace{0^p 1^p 0^p}_{w_1} \underbrace{1^p 0^p 1^p}_{w_2}$$

$$w = 4vxy2$$

$$|vy| \geq 1 \quad |vxy| \leq p$$

$$vxy \in w_1$$

הוכחה 7

$$(p \geq |w_1|)$$

$$|w_1| \geq |w_2|$$

הוכחה 7

$$w \in L$$

$$vxy \in w_2$$

$$w_1 \geq w_2$$

הוכחה 7

$$vxy \in w_2$$

$$w_{i=0} = 0^p 1^p \underbrace{0^p 1^p}_{w_1} 0^p 1^p$$

$$w_{i=0} \in L$$

$$i < 20$$

$$w_{i=0} \in L$$

$$w_{i=0}$$

$$w_{i=0}$$

$$w_{i=0} = w' \bar{w'}$$

$$w' \in L$$

הוכחה 7

$$w' \in L$$

$$w' \in L$$

$$w_{i=0}$$

גלילי $\bar{L}$ P $\rightarrow$ $1/2$

$$L = \{w \mid |w| \bmod 2 = 1\} \cup (L_1 \cup L_2)$$

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$$L_i = \left\{ w \mid \begin{array}{l} \text{מספר } 1 \text{ זוגי} \\ \text{מספר } 0 \text{ זוגי} \\ \text{מספר } 1 \text{ אי-זוגי} \\ |w| \bmod 2 = 0 \end{array} \right\}$$

(גלילי L_1 $\bar{L}$) $\bar{L}$ L_2 $\rightarrow$ $1/2$

גלילי $\bar{L}$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$

~~שאלה~~

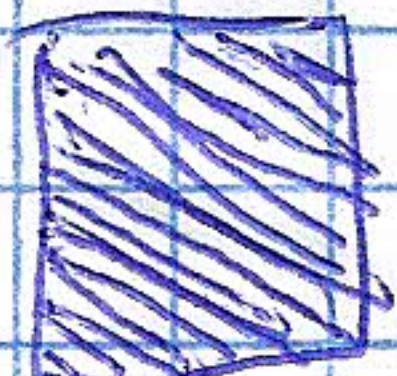
$$S \rightarrow AA$$

$$A \rightarrow 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \mid 0$$

$\rightarrow$ $1/2$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$

$$\{0,1\}^m \cup \{0,1\}^m \cup \{0,1\}^l \cup \{0,1\}^l$$

L $\rightarrow$ $1/2$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$



$m+1$

$2m+1$ $\rightarrow$ $1/2$ $\rightarrow$ $1/2$

$\rightarrow$ $1/2$ $\rightarrow$ $1/2$

$\rightarrow$ $1/2$ $\rightarrow$ $1/2$

$$(m+1) + (m+1) = 2m+2$$

$\rightarrow$ $1/2$ $\rightarrow$ $1/2$

$$L = \{0^{2^k} \mid k \geq 0\}$$

$$w = 0^{2^p}$$

גלילי $1/2$ $\rightarrow$ $1/2$

$$w = 0^{2^p + k}$$

$k \leq p$

$$2^p = |w| < |w_{i=2}| \leq 2^p + p < 2^p + 2^p = 2^{p+1}$$

$$L_2 = \{ a^{3j} b^{2k} \mid j \geq 0 \}$$

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$$L_2 \cap \{ a^i b^j \mid i \geq j \}$$

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$$L_3 = \{ a^{3j} b^{2k} \mid j \geq 1 \}$$

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$$L_3 \cap L_2$$

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$$h(a) = \epsilon \quad h(b) = b$$

$$h(L_3) = b^{2k}$$

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$$u = \epsilon, \quad x = \epsilon, \quad y = \epsilon$$

$$z = \epsilon$$

$$w_i \in L_2$$

$$w_i \in L_2$$

$$w_i \in L_2$$

$$w_i \in L_2$$

$$w_i \in L_2$$

$$w_i = (aaa)^i (aaa)^j$$

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